Light-Front Holography and Gauge/Gravity Duality: The Light Meson and Baryon Spectra

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Starting from the bound state Hamiltonian equation of motion in QCD, we derive relativistic light-front wave equations in terms of an invariant impact variable ζ which measures the separation of the quark and gluonic constituents within the hadron at equal light-front time. These equations of motion in physical space-time are equivalent to the equations of motion which describe the propagation of spin-J modes in anti–de Sitter (AdS) space. Its eigenvalues give the hadronic spectrum, and its eigenmodes represent the probability amplitudes of the hadronic constituents at a given scale. An effective classical gravity description in a positive-sign dilaton background $\exp(+\kappa^2 z^2)$ is given for the phenomenologically successful soft-wall model which naturally encodes the internal structure of hadrons and their orbital angular momentum. Applications to the light meson and baryon spectrum are presented.

1. Introduction

We have recently shown a remarkable connection between the description of hadronic modes in AdS space and the Hamiltonian formulation of QCD in physical space-time quantized on the light-front (LF) at fixed light-front time $\tau = t + z/c$, the time marked by the front of a light wave, [1] rather than at instant time t, the ordinary time. In fact, we can use a first semiclassical approximation to transform the fixed LF time bound-state Hamiltonian equation to a corresponding wave equation in AdS space. To this end, we identify an invariant LF coordinate ζ which allows the separation of the dynamics of quark and gluon binding from the kinematics of constituent spin and internal orbital angular momentum. [2] The result is a single-variable LF relativistic Schrödinger equation which determines the spectrum and LF wavefunctions (LFWFs) of hadrons for general spin and orbital angular momentum. This LF wave equation serves as a semiclassical first approximation to QCD, and it is equivalent to the equations of motion which describe the propagation of spin-J modes on AdS. Remarkably, the AdS equations yield the kinetic energy terms of the partons inside a hadron, whereas the interaction terms build confinement and correspond to truncation of AdS space [2] in an effective dual gravity approximation.

In this note we briefly review the LF quantization of QCD and its LF Fock representation. We derive a relativistic light-front Schrödinger equation from the Hamiltonian bound state equation in light-front QCD following Ref. [2]. The identification of orbital angular momentum of the constituents is a key element in our description of the internal structure of hadrons using holographic principles. In our approach quark and gluon degrees of freedom are explicitly introduced in the gauge/gravity correspondence, in contrast with the usual AdS/QCD framework [3,4] where axial and vector currents become the primary entities as in effective chiral theory. We will also review some of the features of the "hard-wall" [5] and "soft-wall" [6] which provide an initial approximation to QCD. In our approach the holographic mapping is carried out in the strongly coupled regime where QCD is almost conformal, corresponding to an infrared fixed-point. [7] Our analysis follows from recent developments in light-front QCD [8,9,10,11,12] which have been inspired by the AdS/CFT correspondence. [13]

2. Light-Front Quantization of QCD

One can express the hadron four-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, in terms of the dynamical fields, the Dirac field ψ_+, where $\psi_\pm = \Lambda_\pm \psi$, $\Lambda_\pm = \gamma^0\gamma^\pm$, and the transverse field $\mathbf{A}_\perp$ in the $A^+ = 0$ gauge [14]

$$\begin{aligned} P^- &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \, \bar\psi_+ \gamma^+ \frac{m^2 + (i\nabla_\perp)^2}{i\partial^+}\psi_+ \\ &\qquad + (\text{interactions}), \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp \, \bar\psi_+\gamma^+ i\partial^+\psi_+, \\ \mathbf{P}_\perp &= \frac{1}{2}\int dx^- d^2\mathbf{x}_\perp \, \bar\psi_+\gamma^+ i\nabla_\perp \psi_+, \end{aligned} \tag{1}$$

where the integrals are over the initial surface $x^+ = 0$, $x^\pm = x^0 \pm x^3$. The operator P^- generates LF time translations $[\psi_+(x), P^-] = i\frac{\partial}{\partial x^+}\psi_+(x)$, and the generators P^+ and $\mathbf{P}_\perp$ are kinematical. For simplicity we have omitted from (1) the contribution from the gluon field $\mathbf{A}_\perp$.

The Dirac field operator is expanded as

$$\begin{aligned} \psi_+(x^-, \mathbf{x}_\perp)_\alpha &= \sum_\lambda \int_{q^+>0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} \\ &\times \left[b_\lambda(q) u_\alpha(q,\lambda) e^{-iq\cdot x} + d_\lambda(q)^\dagger v_\alpha(q,\lambda) e^{iq\cdot x}\right], \end{aligned} \tag{2}$$

with u and v LF spinors. Similar expansion follows for $\mathbf{A}_\perp$. Using LF commutation relations $\left\{b(q), b^\dagger(q')\right\} = (2\pi)^3 \, \delta(q^+ - q'^+)\delta^{(2)}(\mathbf{q}_\perp - \mathbf{q}'_\perp)$, one has

$$P^- = \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \, \frac{m^2 + \mathbf{q}_\perp^2}{q^+} \, b_\lambda^\dagger(q) b_\lambda(q) + (\text{interactions}). \tag{3}$$

The LF time evolution operator P^- is conveniently written as a term which represents the sum of the kinetic energy of all the partons plus a sum of all the interaction terms.

It is convenient to define a LF Lorentz invariant Hamiltonian $H_{LF} = P_\mu P^\mu = P^-P^+ - \mathbf{P}_\perp^2$ with eigenstates $|\psi_H(P^+, \mathbf{P}_\perp, S_z)\rangle$ and eigenmass $\mathcal{M}_H^2$, the mass spectrum of the color-singlet states of QCD [14]

$$H_{LF}|\psi_H\rangle = \mathcal{M}_H^2|\psi_H\rangle. \tag{4}$$

A state $|\psi_H\rangle$ is an expansion in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian: $|\psi_H\rangle = \sum_n \psi_{n/H}|n\rangle$, where a one parton state is $|q\rangle = \sqrt{2q^+}\, b^\dagger(q)|0\rangle$. The Fock components $\psi_{n/H}(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ are independent of P^+ and $\mathbf{P}_\perp$ and depend only on relative partonic coordinates: the momentum fraction $x_i = k_i^+/P^+$, the transverse momentum $\mathbf{k}_{\perp i}$ and spin component λ_i^z. Momentum conservation requires $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n \mathbf{k}_{\perp i} = 0$. The LFWFs $\psi_{n/H}$ provide a *frame-independent* representation of a hadron which relates its quark and gluon degrees of freedom to their asymptotic hadronic state.

3. Light-Front Holography

We can compute $\mathcal{M}^2$ from the hadronic matrix element

$$\langle\psi_H(P')|H_{LF}|\psi_H(P)\rangle = \mathcal{M}_H^2\langle\psi_H(P')|\psi_H(P)\rangle, \tag{5}$$

expanding the initial and final hadronic states in terms of its Fock components. The computation is simplified in the frame $P = \left(P^+, M^2/P^+, \vec{0}_\perp\right)$ where $H_{LF} = P^+P^-$. We find

$$\mathcal{M}_H^2 = \sum_n \int \left[dx_i\right]\left[d^2\mathbf{k}_{\perp i}\right] \sum_q \Big(\frac{\mathbf{k}_{\perp q}^2 + m_q^2}{x_q}\Big) \times \left|\psi_{n/H}(x_i, \mathbf{k}_{\perp i})\right|^2 + (\text{interactions}), \tag{6}$$

plus similar terms for antiquarks and gluons ($m_g = 0$). The integrals in (6) are over the internal coordinates of the n constituents for each Fock state

$$\int \left[dx_i\right] \equiv \prod_{i=1}^n \int dx_i \, \delta\Big(1 - \sum_{j=1}^n x_j\Big), \tag{7}$$

$$\int \left[d^2\mathbf{k}_{\perp i}\right] \equiv \prod_{i=1}^n \int \frac{d^2\mathbf{k}_{\perp i}}{2(2\pi)^3} \, 16\pi^3 \, \delta^{(2)}\Big(\sum_{j=1}^n \mathbf{k}_{\perp j}\Big), \tag{8}$$

with phase space normalization

$$\sum_n \int \left[dx_i\right]\left[d^2\mathbf{k}_{\perp i}\right] \left|\psi_{n/H}(x_i, \mathbf{k}_{\perp i})\right|^2 = 1. \tag{9}$$

The spin indices have been suppressed. The LFWF $\psi_n(x_i, \mathbf{k}_{\perp i})$ can be expanded in terms of $n-1$ independent position coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$, conjugate to the relative coordinates $\mathbf{k}_{\perp i}$

$$\psi_n(x_j, \mathbf{k}_{\perp j}) = (4\pi)^{(n-1)/2} \prod_{j=1}^{n-1} \int d^2\mathbf{b}_{\perp j} \exp\left(i \sum_{k=1}^{n-1} \mathbf{b}_{\perp k} \cdot \mathbf{k}_{\perp k}\right) \psi_n(x_j, \mathbf{b}_{\perp j}), \quad (10)$$

where $\sum_{i=1}^{n} \mathbf{b}_{\perp i} = 0$. We can also express (6) in terms of the internal coordinates $\mathbf{b}_{\perp j}$ with the result

$$\mathcal{M}_H^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\mathbf{b}_{\perp j} \, \psi_n^*(x_j, \mathbf{b}_{\perp j}) \sum_q \left(\frac{-\nabla^2_{\mathbf{b}_{\perp q}} + m_q^2}{x_q}\right) \psi_n(x_j, \mathbf{b}_{\perp j}) + \text{(interactions)}. \quad (11)$$

The normalization is defined by

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \left|\psi_{n/H}(x_j, \mathbf{b}_{\perp j})\right|^2 = 1. \quad (12)$$

To simplify the discussion we will consider a two-parton hadronic bound state. In the limit of zero quark mass $m_q \to 0$

$$\mathcal{M}^2 = \int_0^1 \frac{dx}{x(1-x)} \int d^2\mathbf{b}_\perp \, \psi^*(x, \mathbf{b}_\perp) \times \left(-\nabla^2_{\mathbf{b}_\perp}\right) \psi(x, \mathbf{b}_\perp) + \text{(interactions)}. \quad (13)$$

The functional dependence for a given Fock state is given in terms of the invariant mass

$$\mathcal{M}_n^2 = \Big(\sum_{a=1}^{n} k_a^\mu\Big)^2 = \sum_a \frac{\mathbf{k}^2_{\perp a} + m_a^2}{x_a} \to \frac{\mathbf{k}^2_\perp}{x(1-x)}\,, \quad (14)$$

the measure of the off-energy shell of the bound state, $\mathcal{M}^2 - \mathcal{M}_n^2$. Similarly in impact space the relevant variable for a two-parton state is $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$. Thus, to first approximation LF dynamics depend only on the boost invariant variable $\mathcal{M}_n$ or ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from the relation

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}, \quad (15)$$

thus factoring out the angular dependence φ and the longitudinal, $X(x)$, and transverse mode $\phi(\zeta)$ with normalization $\langle\phi|\phi\rangle = \int d\zeta \, |\langle\zeta|\phi\rangle|^2 = 1$.

We can write the Laplacian operator in (13) in circular cylindrical coordinates (ζ, φ) and factor out the angular dependence of the modes in terms of the $SO(2)$ Casimir representation L^2 of orbital angular momentum in the transverse plane. Using (15) we find [2]

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta), \quad (16)$$

where all the complexity of the interaction terms in the QCD Lagrangian is summed up in the effective potential $U(\zeta)$. The LF eigenvalue equation $H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle$ is thus a light-front wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta), \quad (17)$$

an effective single-variable light-front Schrödinger equation which is relativistic, covariant and analytically tractable.

It is important to notice that in the light-front the $SO(2)$ Casimir for orbital angular momentum L^2 is a kinematical quantity, in contrast with the usual $SO(3)$ Casimir $\ell(\ell+1)$ from non-relativistic physics which is rotational, but not boost invariant. Using (11) one can readily generalize the equations to allow for the kinetic energy of massive quarks. [15] In this case, however, the longitudinal mode $X(x)$ does not decouple from the effective LF bound-state equations.

As the simplest example, we consider a bag-like model where partons are free inside the hadron

and the interaction terms effectively build confinement. The effective potential is a hard wall: $U(\zeta) = 0$ if $\zeta \leq 1/\Lambda_{\rm QCD}$ and $U(\zeta) = \infty$ if $\zeta > 1/\Lambda_{\rm QCD}$, where boundary conditions are imposed on the boost invariant variable ζ at fixed light-front time. If $L^2 \geq 0$ the LF Hamiltonian is positive definite $\langle \phi | H_{LF} | \phi \rangle \geq 0$ and thus $\mathcal{M}^2 \geq 0$. If $L^2 < 0$ the bound state equation is unbounded from below and the particle "falls towards the center". The critical value corresponds to $L = 0$. The mode spectrum follows from the boundary conditions $\phi(\zeta = 1/\Lambda_{\rm QCD}) = 0$, and is given in terms of the roots of Bessel functions: $\mathcal{M}_{L,k} = \beta_{L,k} \Lambda_{\rm QCD}$. Upon the substitution $\Phi(\zeta) \sim \zeta^{3/2} \phi(\zeta)$, $\zeta \to z$ we find

$$\left[z^2 \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0, \qquad (18)$$

the wave equation which describes the propagation of a scalar mode in a fixed AdS_5 background with AdS radius R. The five dimensional mass μ is related to the orbital angular momentum of the hadronic bound state by $(\mu R)^2 = -4 + L^2$. The quantum mechanical stability $L^2 > 0$ is thus equivalent to the Breitenlohner-Freedman stability bound in AdS. [16] The scaling dimensions are $\Delta = 2 + L$ independent of J in agreement with the twist scaling dimension of a two parton bound state in QCD. Higher spin-J wave equations are obtained by shifting dimensions: $\Phi_J(z) = (z/R)^{-J} \Phi(z)$. [2]

The hard-wall LF model discussed here is equivalent to the hard wall model of Ref. [5]. The variable ζ, $0 \leq \zeta \leq \Lambda_{\rm QCD}^{-1}$, represents the invariant separation between pointlike constituents and is also the holographic variable z in AdS, thus we can identify $\zeta = z$. Likewise a two-dimensional oscillator with effective potential $U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$ is similar to the soft-wall model of Ref. [6] which reproduce the usual linear Regge trajectories, where L is the internal orbital angular momentum and S is the internal spin. As we will show below, the soft-wall discussed here correspond to a positive sign dilaton, and higher-spin solutions follow from shifting dimensions: $\Phi_J(z) = (z/R)^{-J} \Phi(z)$.

The resulting mass spectra for mesons at zero quark mass is $\mathcal{M}^2 = 4\kappa^2(n + L + S/2)$. Thus one can compute the hadron spectrum by simply

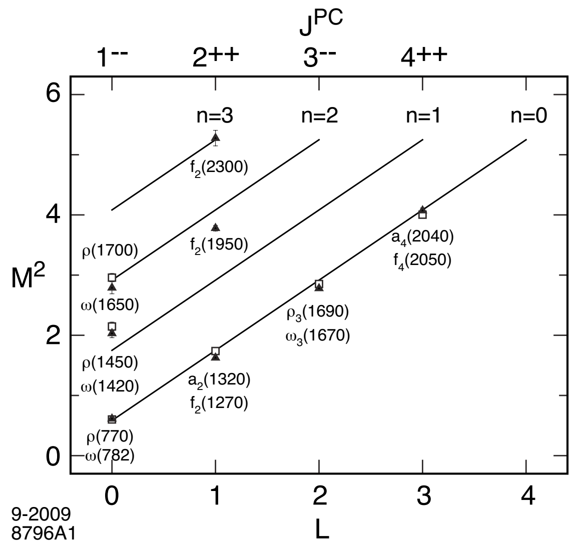

Figure 1. Regge trajectories for the $I=1$ ρ-meson and the $I=0$ ω-meson families for $\kappa = 0.54$ GeV.

adding $4\kappa^2$ for a unit change in the radial quantum number, $4\kappa^2$ for a change in one unit in the orbital quantum number and $2\kappa^2$ for a change of one unit of spin to the ground state value of $\mathcal{M}^2$. Remarkably, the same rule holds for baryons as shown below.

Individual hadron states can be identified by their interpolating operator at $z \to 0$. For example, the pseudoscalar meson interpolating operator $\mathcal{O}_{2+L} = \bar{q}\gamma_5 D_{\{\ell_1} \cdots D_{\ell_m\}} q$, written in terms of the symmetrized product of covariant derivatives D with total internal orbital momentum $L = \sum_{i=1}^m \ell_i$, is a twist-two, dimension $3 + L$ operator with scaling behavior determined by its twist-dimension $2+L$. Likewise the vector-meson operator $\mathcal{O}_{2+L}^\mu = \bar{q}\gamma^\mu D_{\{\ell_1} \cdots D_{\ell_m\}} q$ has scaling dimension $\Delta = 2+L$. The scaling behavior of the scalar and vector AdS modes $\Phi(z) \sim z^\Delta$ at $z \to 0$ is precisely the scaling required to match the scaling dimension of the local pseudoscalar and vector-meson interpolating operators. The spectral predictions for light vector meson states are compared with experimental data in the Chew-

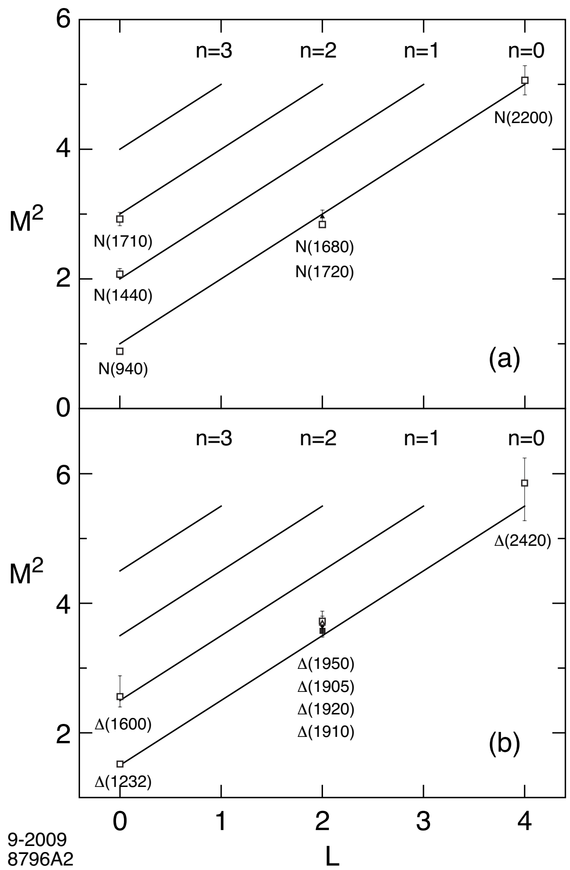

Figure 2. **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV

Frautschi plot in Fig. 1 for the soft-wall model discussed here.

The twist dimension of the interpolating operator ensures dimensional counting rules for form factors and other hard exclusive processes, consistent with conformal invariance at short distances as well as the scaling expected from supersymmetry, [17] since the scalar field, the spinor field and the gluon field G all have twist one.

For baryons, the light-front wave equation is a linear equation determined by the LF transformation properties of spin 1/2 states. A linear confining potential $U(\zeta) \sim \kappa^2 \zeta$ in the LF Dirac equation leads to linear Regge trajectories. [15] For fermionic modes the light-front matrix Hamiltonian eigenvalue equation $D_{LF}|\psi\rangle = \mathcal{M}|\psi\rangle$, $H_{LF} = D_{LF}^2$, in a 2×2 spinor component representation is equivalent to the system of coupled linear equations

$$\begin{aligned} -\frac{d}{d\zeta}\psi_- - \frac{\nu+\frac{1}{2}}{\zeta}\psi_- - \kappa^2\zeta\psi_- &= \mathcal{M}\psi_+, \\ \frac{d}{d\zeta}\psi_+ - \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ - \kappa^2\zeta\psi_+ &= \mathcal{M}\psi_-. \end{aligned} \tag{19}$$

with eigenfunctions

$$\begin{aligned} \psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu}(\kappa^2\zeta^2), \\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2), \end{aligned} \tag{20}$$

and eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1). \tag{21}$$

The baryon interpolating operator $\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi$, $L = \sum_{i=1}^m \ell_i$, is a twist 3, dimension $9/2 + L$ with scaling behavior given by its twist-dimension $3 + L$. We thus require $\nu = L + 1$ to match the short distance scaling behavior. Higher spin fermionic modes are obtained by shifting dimensions for the fields as in the bosonic case. Thus, as in the meson sector, the increase in the mass squared for higher baryonic states is $\Delta n = 4\kappa^2$, $\Delta L = 4\kappa^2$ and $\Delta S = 2\kappa^2$, relative to the lowest ground state, the proton.

The predictions for the **56**-plet of light baryons under the $SU(6)$ flavor group are shown in Fig. 2. As for the predictions for mesons in Fig. 1, only confirmed PDG [18] states are shown. The Roper state $N(1440)$ and the $N(1710)$ are well accounted for in this model as the first and second radial states. Likewise the $\Delta(1660)$ corresponds to the first radial state of the Δ family. The model is successful in explaining the important parity degeneracy observed in the light baryon spectrum, such as the $L = 2$, $N(1680) - N(1720)$ pair and the $\Delta(1905), \Delta(1910), \Delta(1920), \Delta(1950)$ states which are degenerate within error bars. Parity degeneracy of baryons is also a property of the hard wall model, but radial states are not

well described by this model. [9] Recent work on the hadronic spectrum based on AdS/QCD models is given in [19,20,21,22,23,24,25,26].

4. A Soft-Wall Model

A nonconformal metric dual to a confining gauge theory is conveniently written as [5]

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right), \qquad (22)$$

where the warp factor $A(z) \to 0$ at small z for geometries which are asymptotically AdS_5. According to Sonnenscheim [27] a background dual to a confining theory should satisfy the conditions for the time-time metric component g_{00}

$$\partial_z(g_{00})|_{z=z_0} = 0, \qquad g_{00}|_{z=z_0} \neq 0, \qquad (23)$$

to display Wilson loop area law for confinement of strings.

We consider a warp factor $2A(z) = \pm\kappa^2 z^2$ [28, 29] which, for our present discussion can be considered similar to the dilaton background introduced in Ref. [6]. The metric for the positive sign warp factor $2A(z) = +\kappa^2 z^2$ satisfy the conditions (23) with $z_0 = 1/\sqrt{2}\kappa$. This type of solution was considered in [29] to derive a confining potential between heavy quarks. The area law confining conditions (23) are not obeyed for the negative sign warp factor, $A(z) = -\kappa^2 z^2$, the solution described in [6].

We may also study the confinement properties of the warped metrics by following an object in AdS space as it falls to the infrared region by the effects of gravity. The gravitational potential energy for an object of mass m in general relativity is given in terms of g_{00}

$$V = mc^2\sqrt{g_{00}} = mc^2 R\,\frac{e^{A}(z)}{z}. \qquad (24)$$

For the negative solution the potential decreases monotonically, and thus an object in AdS will fall to infinitely large values of z. For the positive solution, the potential is nonmonotonic and has an absolute minimum at $\bar{z} = 1/\kappa$. For large values of z the gravitational potential increases exponentially, thus confining any object in the modified AdS metrics to distances $\langle z\rangle \sim 1/\kappa$. Indeed the potential (24) for a positive sign factor $2A(z) = +\kappa^2 z^2$ is similar to the confining potential depicted in Ref. [30]. We will study here the positive sign solution, which as discussed above, has an interesting physical motivation. [31]

For practical reasons we introduce a dilaton background $\varphi(z)$ as in Ref. [6], instead of deforming the AdS_5 metrics, and write the action

$$S = \int d^d x\, dz\, \sqrt{g}\, e^{\varphi(z)} \mathcal{L}, \qquad (25)$$

in AdS_{d+1} for the dilaton field $\varphi(z) = \pm\kappa^2 z^2$. For a scalar field the Lagrangian $\mathcal{L} = \frac{1}{2}\left(g^{\ell m}\partial_\ell\Phi\partial_m\Phi - \mu^2\Phi^2\right)$ leads to the wave equations

$$\left[z^2\partial_z^2 - \left(d - 1 \mp 2\kappa^2 z^2\right) z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi(z) = 0 \qquad (26)$$

with $(\mu R)^2 \geq -4$. For $\varphi(z) = -\kappa^2 z^2$ there is no stable solution for the lowest value $(\mu R)^2 = -4$. For $\varphi(z) = \kappa^2 z^2$ the lowest possible solution corresponding to $(\mu R)^2 = -4$ is $\Phi(z) \sim z^2 e^{-\kappa^2 z^2}$, with eigenvalue $\mathcal{M}^2 = 0$. This is a chiral symmetric bound state of two massless quarks with scaling dimension 2 and size $\langle z^2\rangle \sim 1/\kappa^2$, which we identify with the lowest state, the pion.

We define a spin-J mode $\Phi_{\mu_1\cdots\mu_J}$ with all indices along 3+1 with shifted dimensions $\Phi_J(z) = (z/R)^{-J}\Phi(z)$ and normalization

$$R^{d-2J-1}\int_0^\infty \frac{dz}{z^{d-2J-1}}\, e^{\kappa^2 z^2}\, \Phi_J^2(z) = 1. \qquad (27)$$

The shifted field Φ_J obeys the wave equation

$$\left[z^2\partial_z^2 - \left(d{-}1{-}2J - 2\kappa^2 z^2\right) z\,\partial_z + z^2\mathcal{M}^2 - (\mu R)^2\right]\Phi_J = 0 \qquad (28)$$

which follows from (26) upon mass rescaling $(\mu R)^2 \to (\mu R)^2 - J(d-J)$ and $\mathcal{M}^2 \to \mathcal{M}^2 - 2J\kappa^2$.

Upon the substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2\zeta^2/2}\,\Phi_J(\zeta)$, we find for $d = 4$ the QCD light-front wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(L+S-1)\right)\phi_{\mu_1\cdots\mu_J} = \mathcal{M}^2\phi_{\mu_1\cdots\mu_J}, \qquad (29)$$

where $J_z = L_z + S_z$ and $(\mu R)^2 = -(2-J)^2 + L^2$.

Equation (29) has eigenfunctions

$$\phi_n^L(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2), \tag{30}$$

and eigenvalues

$$\mathcal{M}_{n,L,S}^2 = 4\kappa^2\left(n + L + \frac{S}{2}\right). \tag{31}$$

We thus recover the potential $U(z) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$ used above for the computation of the meson spectrum.

5. Conclusions

We have derived a connection between a semiclassical first approximation to QCD quantized on the light-front and hadronic modes propagating on a fixed AdS background. This duality leads to a remarkable Schrödinger-like equation in the AdS fifth dimension coordinate z (17). We have shown how the identical AdS wave equation can be derived in physical space time as an effective equation for valence quarks in LF quantized theory, where one identifies the AdS fifth dimension coordinate z with the LF coordinate ζ. We originally derived this correspondence using the identity between electromagnetic and gravitational form factors computed in AdS and LF theory [10,11,12]. Our derivation also shows that the mass μ is directly related to orbital angular momentum L in physical space-time. The result is physically compelling and phenomenologically successful.

We have shown how the soft-wall AdS/CFT model with a dilaton-modified AdS space leads to the potential $U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$. This potential can be derived directly from the action in AdS space and corresponds to a dilaton profile $\exp(+\kappa^2 z^2)$, with opposite sign to that of Ref. [6]. Hadrons are identified by matching the power behavior of the hadronic amplitude at the AdS boundary at small z to the twist of its interpolating operator at short distances $x^2 \to 0$, as required by the AdS/CFT dictionary. The twist corresponds to the dimension of fields appearing in chiral super-multiplets. The twist of a hadron equals the number of constituents.

The light-front AdS/QCD equation provides remarkably successful predictions for the light-quark meson and baryon spectra, as function of hadron spin, quark angular momentum, and radial quantum number. [32] The pion is massless, corresponding to zero mass quarks, in agreement with chiral invariance arguments. The predictions for form factors are remarkable successful. The predicted power law fall-off agrees with dimensional counting rules as required by conformal invariance at small z. [11,15]

Higher spin modes follow from shifting dimensions in the AdS wave equations. In the hard-wall model the dependence is linear: $\mathcal{M} \sim 2n + L$. However, in the soft-wall model the standard Regge behavior is found $\mathcal{M}^2 \sim n + L$. Both models predict the same multiplicity of states for mesons and baryons, which is observed experimentally. [33] As in the Schrödinger equation, the semiclassical approximation to light-front QCD described in this paper does not account for particle creation and absorption; it is thus expected to break down at short distances where hard gluon exchange and quantum corrections become important. However, one can systematically improve the semiclassical approximation, for example by introducing nonzero quark masses and short-range Coulomb corrections to describe the dynamics of heavy and heavy-light quark systems.

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Added Note

Following this article a preprint by Fen Zuo appeared (arXiv:0909.4240) where it is shown that the introduction of a positive dilaton profile is particularly suited for describing chiral symmetry breaking. In contrast with the original model [6], the expectation value of the scalar field associated with the quark mass and condensate does not blow up in the far infrared region of AdS.

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